The Ionian Greek philosopher Heraclitus believed that there was a kind of harmony of all moving bodies, known as Logos. Later the Greek philosopher Aristotle taught that the reason that heavy elements, such as Earth move downward is because their natural state is to be in the center of the Earth, while the natural state of light elements, such as fire was to move up towards the inner surface of the sphere of the moon. Archimedes postulated that the center of gravity between two equal weights would be located in the line that joins them. Vitruvius, the roman engineer and architect postulated that gravity depended on the nature or specific gravity of a substance, and used this as the explanation for why some substances sink below other substances. The Alexandrian Scholar John Philoponus proposed the impetus hypothesis that a moving object has a causative force that decreases with time.

Brahmagupta, an Indian astronomer was the first person to describe gravity as an attractive force, using the term "gurutvakarsanam within the heliocentric view of the solar system. He distinguished between external force and inclination or mayl, and proposed that an object gets mayl when in opposition to its natural state of motion. He believed that the Earth was the same on all sides, and that it was the nature of the Earth to attract, and keep things.

Ibn Sina agreed with Philoponus agreed with the idea of impetus, but thought that an external force was required to decrease it. He argued that an object stays in motion until its mayl is spent. The polymath Al-Birundi proposed that heavenly bodies have mass and gravity like the Earth. Al-Khazini proposed that the gravity of a body can vary depending on how far away it is from the center of the universe. Al-Birundi, and Al-Khazini both studied the theory of the center of gravity, and applied it to three dimensional objects. They created the science of gravity, and developed experimental methods for determining the specific gravity of objects, based on the theory of balance and weight. Abu'l-Barakat al-Baghdadi proposed that a mover imparts a violent inclination on a moved, which diminishes as the moved gets further away from the mover. He proposed that an falling objects accelerate with time. Ibn Bajjah proposed that there is a reaction force for every force. Al-Birjandi proposed a hypothesis similar to Galileos notion of circular inertia, which he used to try to explain planetary orbits without gravity.

The French philosopher Jean Buridan and the Merton College of Oxford both attributed the motion of objects to an impetus, which varied depending on the objects mass and velocity. Buridan and Albert of Saxony adopted Abu'l-Barakat's proposal that an increasing impetus is the cause of the acceleration of a falling body. Albert developed a square developed a square law regarding the relationship between the speed of an object in free fall and the time or space elapsed, and proposed that mountains and valleys were caused by erosion, displacing the center of gravity of the Earth. The Merton College developed the mean speed theorem, which would be influential in later gravitational equations. Leonardo da Vinci believed that the origin of gravity was energy. He associated gravity with water and earth. Nicolaus Copernicus wrote about the heliocentric model, in which he had the Earth in the center of the orbit of the Moon. By 1544 experiments from at least two Italians had shown that the weight of objects doesn't affect how they fall. Domingo suggested that an object in free fall accelerates at a constant rate. Galileo proposed that the distance traveled by a falling object would be proportional to the square of the time passed. He also suggested that air resistance was the reason that some objects appear to fall at different rates.

Isaac Newton proposed that currents in the aether streams were the cause of gravity. He proposed that the aether was less dense near massive bodies, and that this would cause a forces between the bodies.

Isaac Newton proposed that the gravity from a body would drop off proportional to the inverse square of the distance. He gave the equation for the force between two bodies as

$$F_g \propto \frac{M_1 M_2}{r^2}$$

Albert Einstein developed his special theory of Relativity in 1905. In 1913 Nordstrom developed the first self-consistent relativistic theory of gravity. Einstein developed the idea that gravity was equivalent to acceleration between 1911 and 1915.

When I was in high school I became interested in higher dimensions. I found that I could figure out some things about a fourth spatial dimension by visualizing the fourth dimension using time as the fourth dimension, but this took a tremendous amount of concentration to control the images in my mind enough to extract useful information from them, and only really worked when the images required could be very simple.

I thought that it would be nice to live in a world with four spatial dimensions as then I wouldn't have to worry about crossing roads as I could just go around roads, and also noticed that three dimensions was too few to extrapolate how the number of regular shapes changes between the number of dimensions. Also if I lived in higher dimensions could allow for a more space, allowing for a more complex world, and allowing me to have a more complex brain.

I looked up the fourth dimension shortly after getting into college, and found a site that talked about what it would be like in two dimensions and four dimensions. Some of it was stuff that I either already knew or could have figured out, but I also found a forum, and found that some of the things people talked about were things I didn't already know. For instance I read that there were six regular polytopes in four dimensions, and three in all dimensions greater than four dimensions. Also I read some posts from users who said that they found that stable orbits were impossible in four spatial dimensions, which surprised me, as I hadn't considered that orbits partly because I didn't know enough physics to know that the motion of bodies in orbit came could be deduced from other laws of physics, and just assumed that planetary motion was just a fundamental law of physics. I also was surprised to read that atoms were unstable in four dimensions and higher.

I noticed that in physics a square comes up in some equations, and because I didn't know the derivation for a lot of the equations came from I thought that anytime I saw a square it might come from us living in three spatial dimensions. I knew that the inverse square law in gravity comes from living in three spatial dimensions, and thought that the velocity square terms in the equations $a = \frac{v^2}{r}$ and $KE = \frac{1}{2}mv^2$ might also come from living in three dimensions of space. I also contemplated whether there would be any way for a universe with four spatial dimensions to have stable orbits, and stable atoms but didn't know enough physics to really know how the laws of physics could or couldn't be adjusted, and didn't know how to figure out whether any particular adjustment would allow for stable orbits or stable atoms.

Eventually I saw a post from someone mentioning a site where it was possible to change the force law and that no matter what he/she did he/she could not get stable orbits, so I decided to check the site out myself, and found that I also could not get stable orbits using the inverse cube law no matter how hard I tried. I decided to try some inverse laws between the inverse square law and inverse cube law, and found that it appeared that even inverse laws very close to the inverse cube law still had stable orbits. Also when the force did not depend on distance, and when the force would increase with distance stable orbits would be possible.

I found that while some force laws, such as the inverse laws with the distance being raised to a power greater than three did not allow for stable orbits some such as the inverse l law allow for stable orbits, and the ones that allowed for stable orbits seemed to also allow for solar systems and moons.

I also saw a post on the site talking about interesting force laws, including different inverse and reciprocal laws, and some force laws involving trigonometric functions, although at the time it was only inverse and reciprocal laws that I was interested in.

I also learned in Introductory Physics that acceleration is the rate of change in position, and that force is the acceleration multiplied by the mass. I also learned that the final position of an object that is accelerating at a constant rate is given by $x_f = \frac{1}{2}at^2 + v_0 t + x_0$ with t being the time passed, x being the position, v being the velocity, a being the acceleration, the subscript 0 denoting initial, and the subscript f denoting final. Also the final velocity of an object accelerating at a constant rate is given by the equation $v_f = at + v_0$. I also learned that the equations of motion for a spring involved sine and cosine, and I also learned that the force acting on an object attached to the end of a spring increases in proportion to how far away it is from what the spring is attached to. I also learned that position, velocity, acceleration, and force are vectors and so the way to find the total force was to add up the components of the different force vectors.

At the same time that I was learning introductory physics I was also taking Calculus, where I learned about derivatives. I learned that derivatives described the slope of a curve at a particular point. I learned that the derivative of x^n is nx^{n-1}, and that $(f + g)' = f' + g'$. I also learned that $sin' x = cosx$ and $cos' x = - sinx$. I also learned that $a \cdot f' = (a \cdot f)'$.

I also learned that velocity is the time derivative of position, and acceleration is the time derivative of velocity. From this I could figure out that the $\frac{1}{2}$ in the equation $x_f = \frac{1}{2}at^2 + v_0 t + x_0$ came from the equations $\left(x^2\right)' = 2x$ and $v_f = at + v_0$.

I also wasn't sure what the analog of changing the force law would be for a quantum system. I also tried simulating orbits when the force was proportional to e^-r, and found that this produced stable orbits.

Eventually I tried the force law (r^-3) and found that the simulation produced stable orbits for this force law, but not for r^-3 even though both force laws were equivalent, and asked in the forum why that was the case. I found out that the problem was that I did not have the downloadable version, and so the creator of the site gave me the downloadable version for free.

Once I had the downloadable version I started trying a variety of force laws including force laws involving trigonometric functions, and a force law involving the force depending on the natural log of the distance. I found that force laws such as sin(r), cos(r), tan(r), sec(r), csc(r), and cot(r) all allowed for stable orbits in the form of star shaped orbits. I also found that (e^-r)*(r^-3) did not allow for stable orbits, implying that r^-3 takes precedent to e^-r. I also found that multiplying r^-3 by a trigonometric function would produce stable orbits implying that trigonometric functions take precedent to r^-3. One found that it seemed that every force law, that I tried, in which gravity was attractive at some distances and repulsive at other distances would produce stable orbits. I tried giving a planet a moon in at least one of them and found that the planet could hold onto a moon.

I wondered also if there was a way to simulate atomic orbitals, and change the laws of physics, but I found that the concept of force is not used in quantum physics, and was not sure what the closest analog would be to changing a force law in quantum physics. I also found that I was unsure what equations in quantum physics depend on the number of dimensions, and which ones do not.

I also wondered if changing the laws of motion would allow for stable orbits, and was also wanting to simulate the case, in which inertial and gravitational mass would not be equivalent. I tried suggesting these to the creator of the site, and he said that it might get too complicated to add, and that different inertial and gravitational mass might be too far from real physics.

I began to try to figure out how to try to figure out how to create my own simulator, and at first tried to see if I could work out how to create one using the same program that the site I was using used, but my programming knowledge was far from advanced enough for me to work out how to create any type of physics simulation in that program. Also for a force law $F=f(x)$ the force is the second time derivative of r is proportional to $f(x)$ considering that force is the second derivative of position. I also found that all of the functions I tried were not solutions to the second order differential equations I was interested in, and I was not able to understand differential equations well enough to find the equations of motion for bodies in orbit.

I also found that in the information page for what Greg Egan calls the Riemannian Universe, in which the spacetime interval between two events is

$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 + \Delta w^2$ and which is described in his Orthogonal Series says that the electric field around a charge is proportional to $-\frac{\cos cos\,(br) + br\sin sin\,(br)}{r^2}$, with b being related to the mass of the photon, implying that the electric force between two electric charges would also be proportional to $-\frac{\cos cos\,(br) + br\sin sin\,(br)}{r^2}$, meaning that it would be attractive at some distances and repulsive at other distances. Also for two like charges the force would be attractive at the closest distances. Given that all forces I had previously tested, in which the direction of the force depends on distance allowed for stable orbits, and the conjecture that if the direction of the force depended on distance in what Greg Egan called the Riemannian Universe, I conjectured that in a universe that was similar to the Riemannian Universe, but with more dimensions it would be possible for two electric charges to orbit each other provided that they were far enough away.

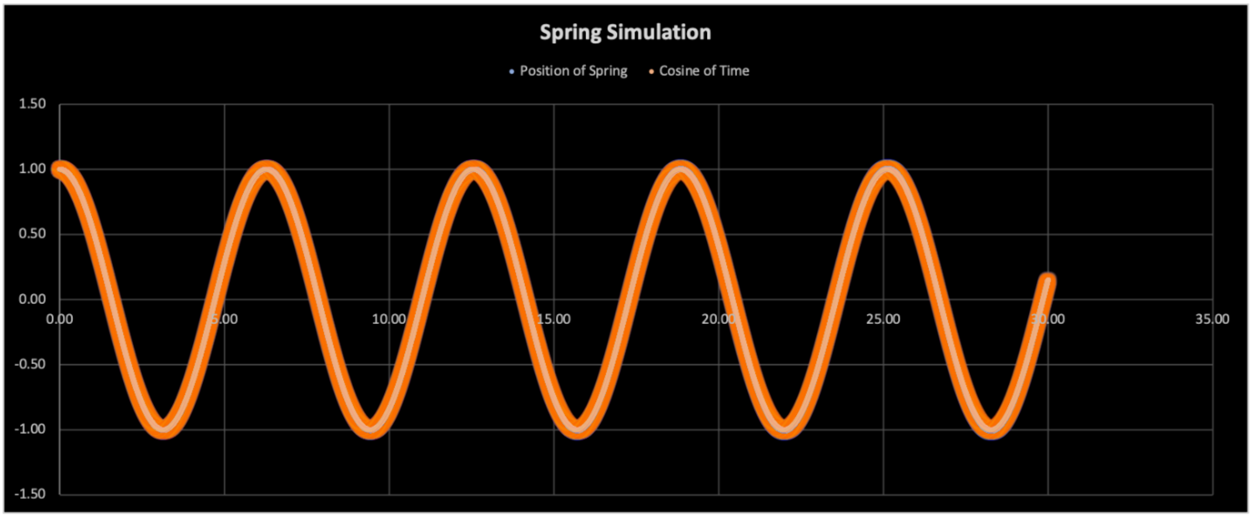
I simulated what I suspected might be the force law for the electric force a universe similar to the one Greg Egan was describing, but with an extra dimension, and found that I could get an entire solar system of 7 planets to remain in what seemed to be stable orbits. Also I had a moon in a meta stable orbit.

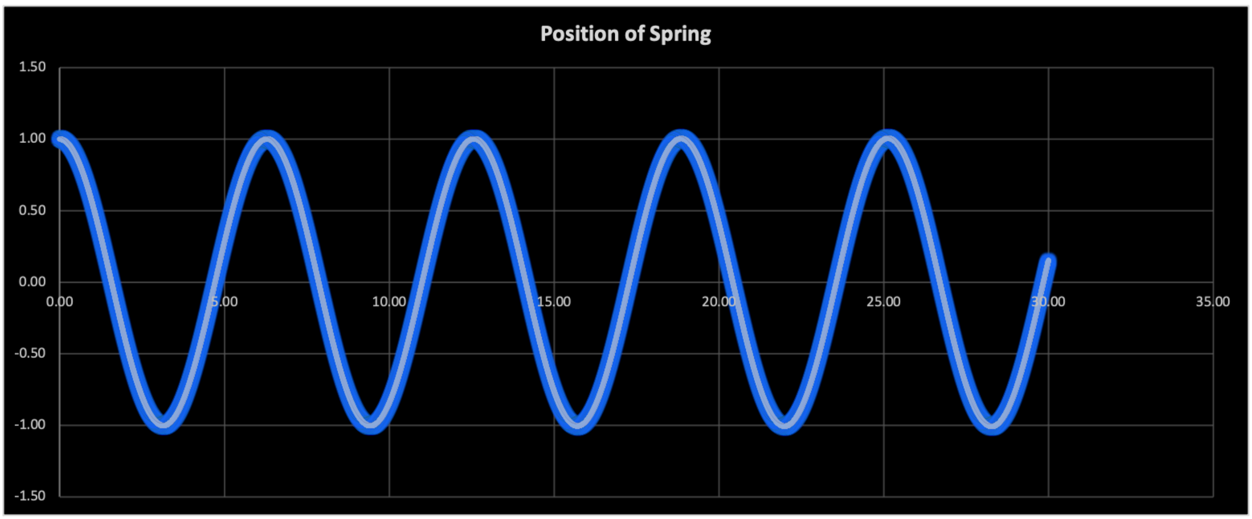
I wondered whether it might be possible for two planets to orbit a star at the same radius, but on different orbital planes, but I did not know of a simulator that would allow me to simulate such a scenario, as the simulator I had been using only allowed me to simulate orbits in 2 dimensions.

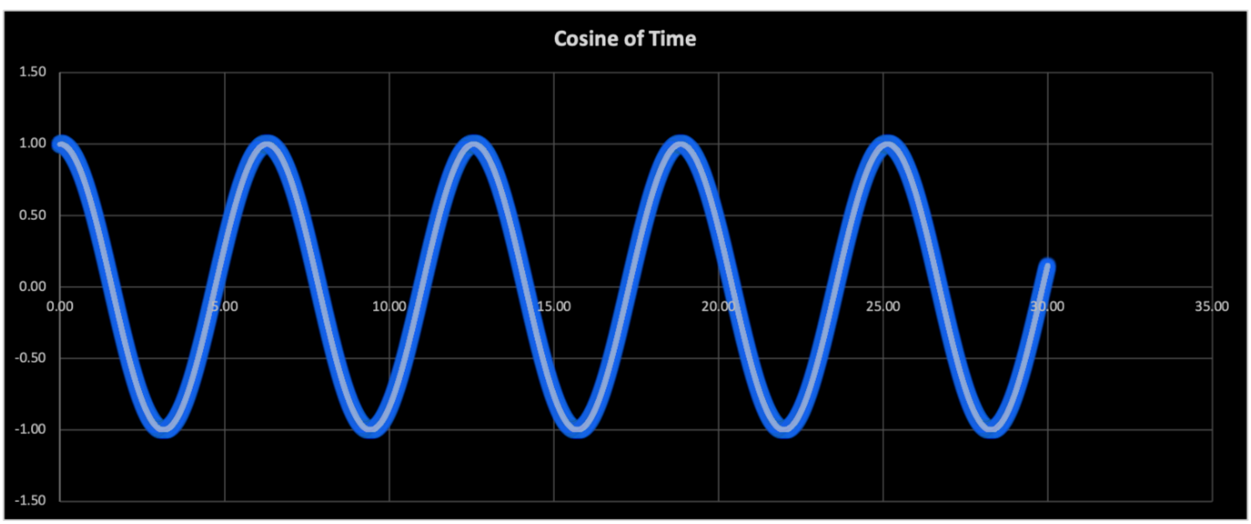
I realized that I might be able to approximate the equations of motion for the case that the acceleration of two bodies depends on distance, using the equations of motion for the case of constant acceleration $x_f = \frac{1}{2}at^2 + v_0 t + x_0$ and $v_f = at + v_f$, the equation $F = f(r)$ and the equation $a = \frac{F}{m}$. I already knew how to use formulas in excel, as well as how to create recursive functions, and so I decided to start creating two body simulations. Initially I got ahead of myself by trying to simulate force laws and laws of motion, that were unavailable in the simulation from the site I had used before, before realizing that I should start by simulating using force laws, in which I already knew how the bodies should move in order to make sure that was using the correct formulas, and that my time step was small enough for an accurate simulation. I also realized that I should start by simulating motion in one dimension before moving onto motion in two or more dimensions.

I started trying to simulate the motion of a spring, and noticed that the simulation seemed to give the wrong results. I found that the problem was that in order to calculate each new value of x I was referencing the initial time, and once I started referencing the time step and adjusting the size of the time step. In my simulation I started using the formulas $x(t + \Delta t) = \frac{1}{2}a(t) * (\Delta t)^2 + v(t) * \Delta t + x(t)$ and $v(t + \Delta t) = a(t) * \Delta t + v(t)$ and $a(t) = x(t)$. I used different columns for time step increments, time passed, position, velocity, and acceleration, and had time as being vertical.

Below are graphs of the position of a spring through time and the cosine of the time passed, which was produced using simulations I created in excel.


Spring Simulation
Position of Spring
Cosine of Time
1.50
1.00
0.50
0.00
-0.50
-1.00
-1.50
0.00
5.00
10.00
15.00
20.00
25.00
30.00
35.00


Position of Spring
1.50
1.00
0.50
0.00
-0.50
-1.00
-1.50
0.00
5.00
10.00
15.00
20.00
25.00
30.00
35.00


Cosine of Time
1.50
1.00
0.50
0.00
-0.50
-1.00
-1.50
0.00
5.00
10.00
15.00
20.00
25.00
30.00
35.00

After I simulated motion in one dimension I began attempting to simulate orbits using the inverse square law in two dimensions. Setting $G =- 1$ the inverse square law for Gravity is $F =- \frac{M_1 M_2}{r^2}$. I used cartesian coordinates in my simulation meaning that r= $\sqrt{\left(x_1 - x_2\right)^2 + \left(y_1 - y_2\right)^2}$ so that the equation becomes $F =- \frac{M_1 M_2}{\left(\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}\right)^2}$. The force on each body is in the direction towards the other body. In order to get the force on each body to be in the direction towards the other body I can multiply the previous equation by the differences between coordinates of each body, and then to keep the same magnitude for the force I can divide by the distance between the two bodies. This means that the forces are described by the equations

$$F_{1x} =- \frac{x_1-x_2}{\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}} \frac{M_1 M_2}{\left(\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}\right)^2}$$

$$F_{1y} =- \frac{y_1-y_2}{\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}} \frac{M_1 M_2}{\left(\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}\right)^2}$$

$$F_{2x} =- \frac{x_2-x_1}{\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}} \frac{M_1 M_2}{\left(\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}\right)^2}$$

$$F_{2y} =- \frac{y_2-y_1}{\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}} \frac{M_1 M_2}{\left(\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}\right)^2}$$

From this I can get the equations for acceleration as

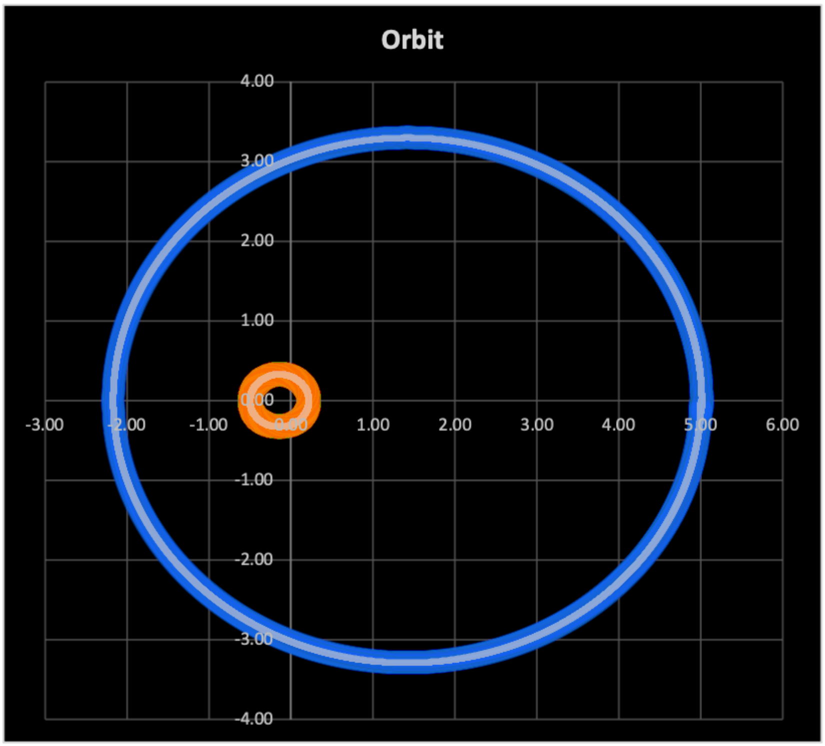
$$a_{1x} =- \frac{x_1-x_2}{\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}} \frac{M_2}{\left(\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}\right)^2}$$

$$a_{1y} =- \frac{y_1-y_2}{\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}} \frac{M_2}{\left(\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}\right)^2}$$

$$a_{2x} =- \frac{x_2-x_1}{\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}} \frac{M_1}{\left(\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}\right)^2}$$

$$a_{2y} = -\frac{y_2 - y_1}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}} \frac{M_1}{\left(\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}\right)^2}$$

Below is a chart of what the orbit of two bodies using the inverse square law in a simulation I created in excel. For this one I chose a time step such that there would be approximately 30,000 time steps per orbit.



Generalizing to force laws F_g=f(r) the equations for acceleration are

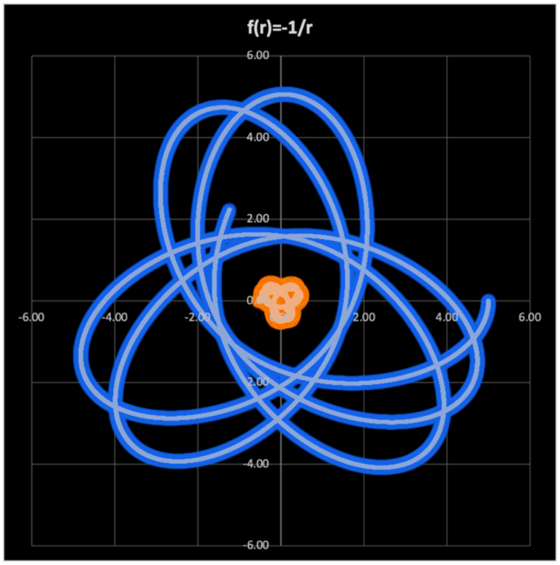
$$a_{1x} = \frac{x_1 - x_2}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}} M_2 f\left(\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}\right)$$

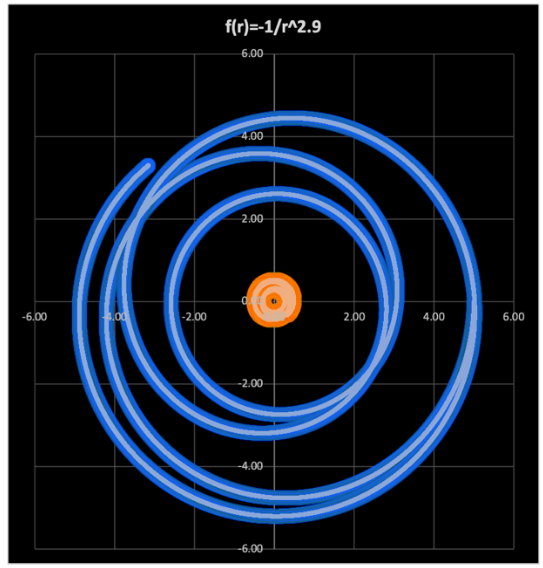
$$a_{1y} = \frac{y_1 - y_2}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}} M_2 f\left(\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}\right)$$

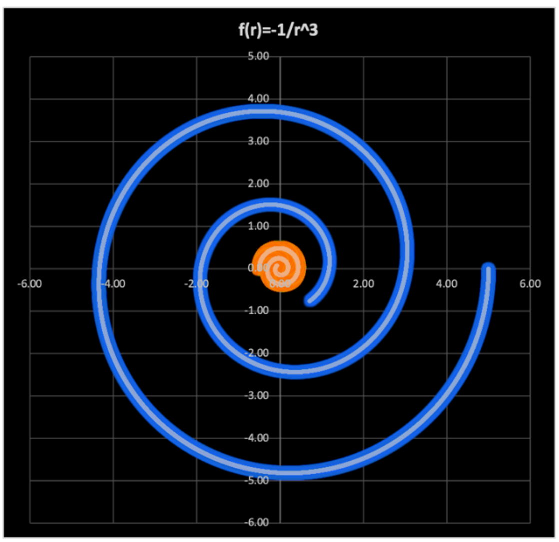
$$a_{2x} = \frac{x_2 - x_1}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}} M_1 f\left(\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}\right)$$

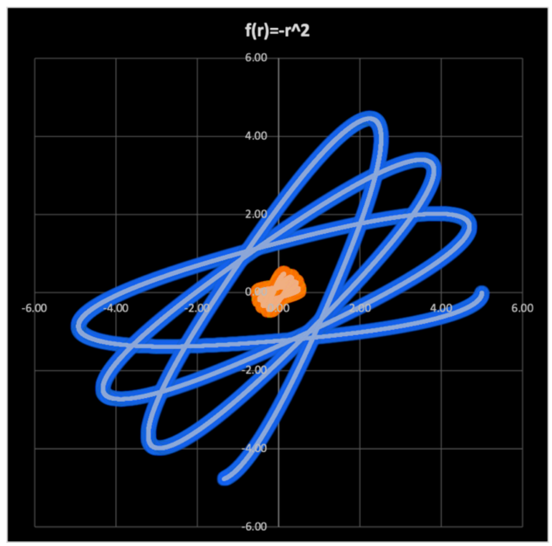
$$a_{2y} = \frac{y_2 - y_1}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}} M_1 f\left(\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}\right)$$

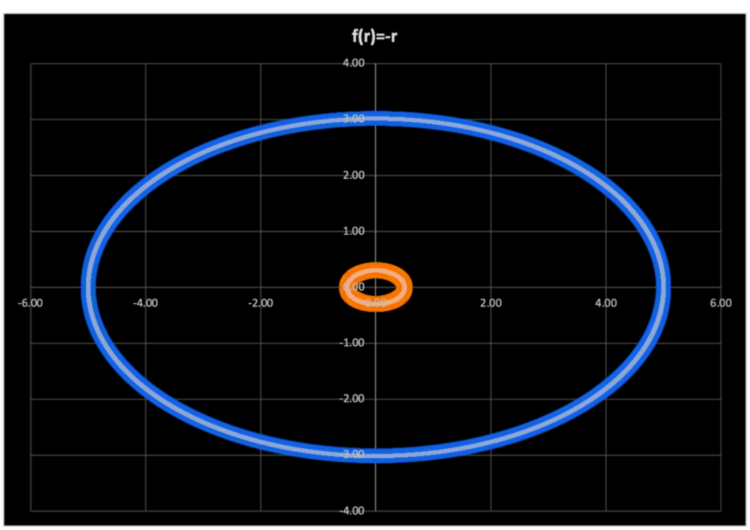
Below is are charts from some simulations I ran in excel using inverse and reciprocal laws in excel.


f(r)=-1/r
6.00
4.00
2.00
-6.00
-4.00
-2.00
2.00
4.00
6.00
-2.00
-4.00
-6.00

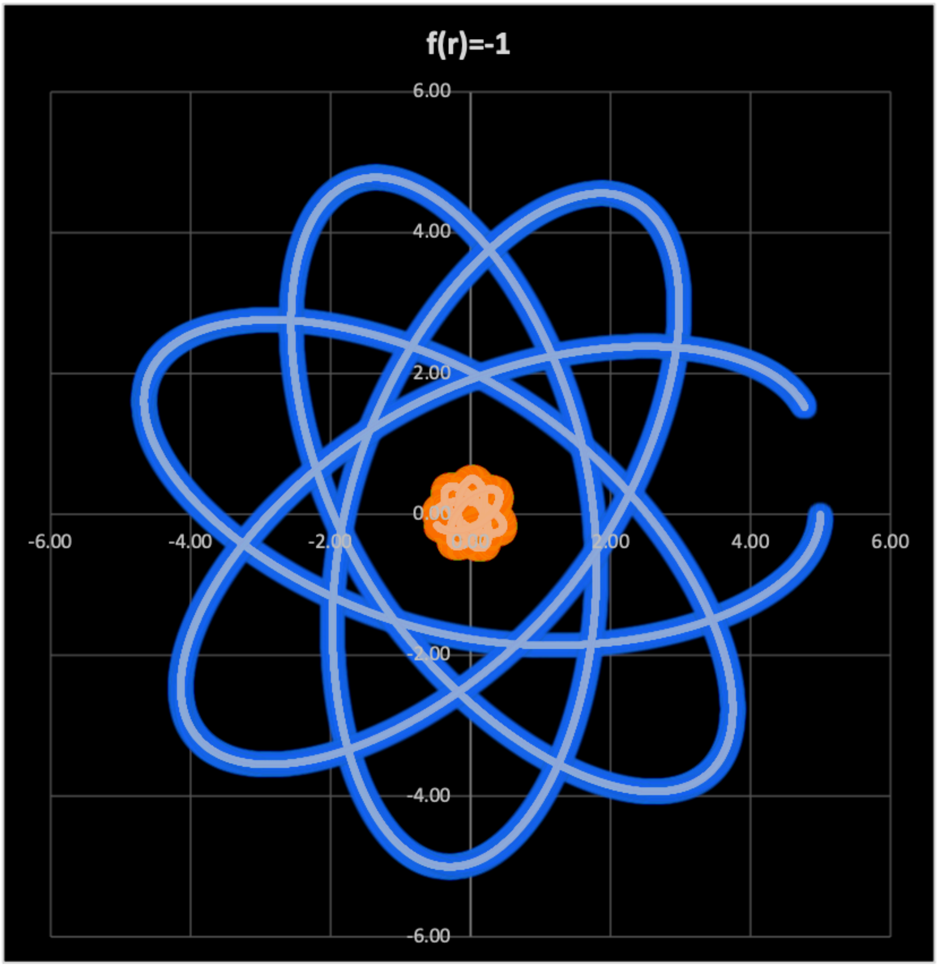

f(r)=-1/r^2.9
6.00
4.00
2.00
-6.00
-4.00
-2.00
2.00
4.00
6.00
-2.00
-4.00
-6.00


f(r)=-1/r^3
5.00
4.00
3.00
2.00
1.00
-6.00
-4.00
-2.00
2.00
4.00
6.00
-1.00
-2.00
-3.00
-4.00
-5.00
-6.00

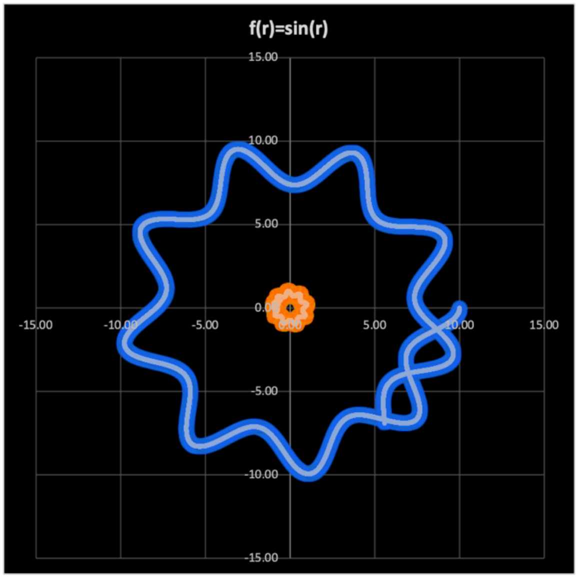

f(r)=-r^2
6.00
4.00
2.00
-6.00
-4.00
-2.00
2.00
4.00
6.00
-2.00
-4.00
-6.00

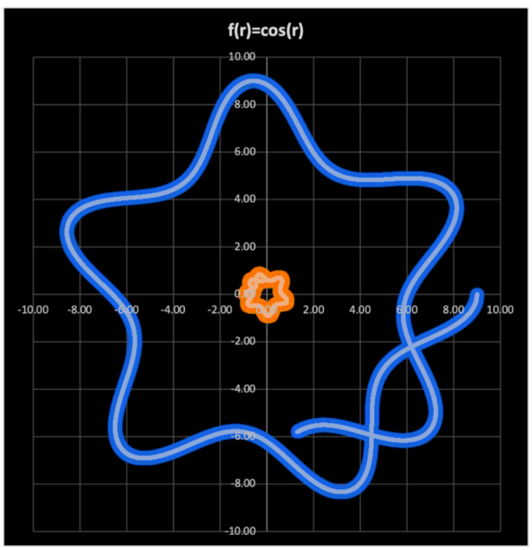

f(r)=-r
4.00
3.00
2.00
1.00
-6.00
-4.00
-2.00
2.00
4.00
6.00
-1.00
-2.00
-3.00
-4.00

Below is a chart from a simulation with the force being the same regardless of distance

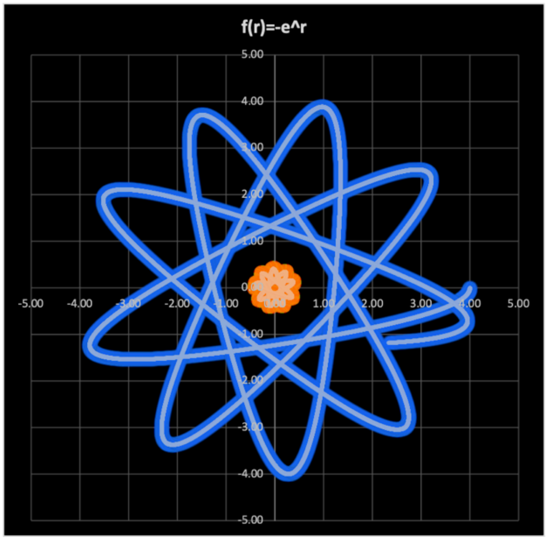


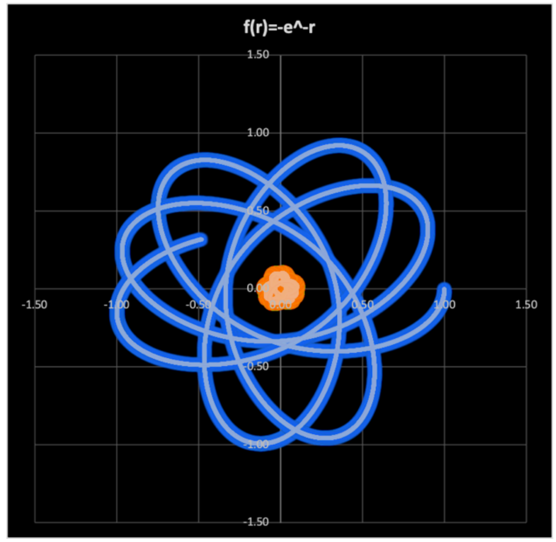
Below are charts from simulations I made in excel, in which the force depends on the sine and cosine of the distance.

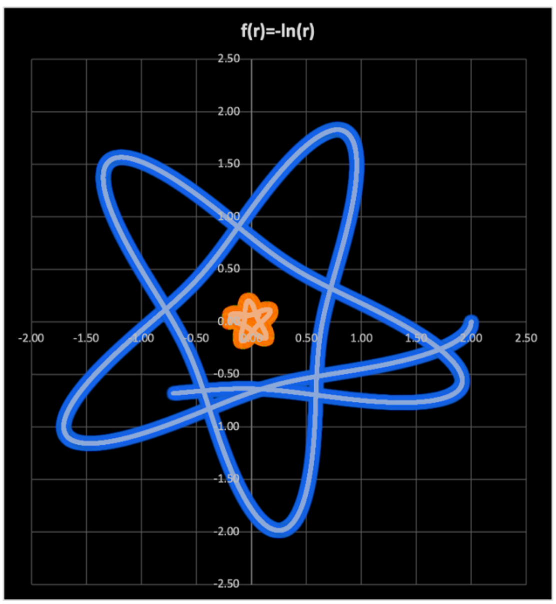




Below are the charts from some simulations I created, in which the forces are exponential and logarithmic functions of distance.





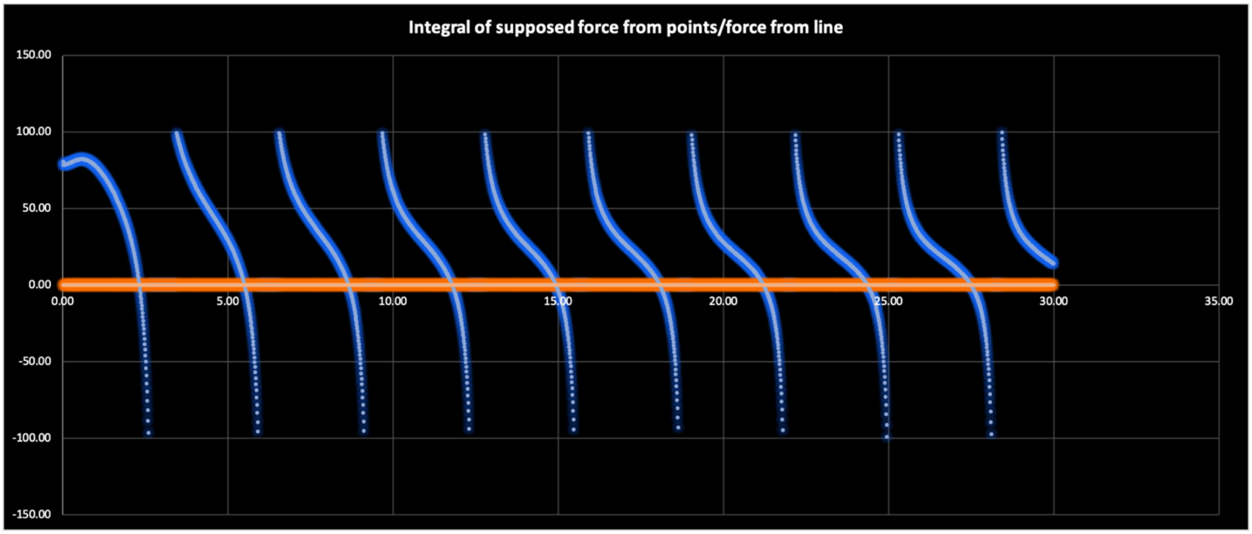


I at first thought that the force law for the force between two electric charges in the universe Greg Egan described in the universe he described as the Riemannian Universe, but with an additional dimension would be similar to the one Greg Egan implied in his Riemannian Universe but with the r^2 in the denominator replaced with an r^3. I initially tried simulating the motion of multiple bodies using this force law, and later thought about how to simulate motion through spacetime using this type of force law.

For the sake of generality let's say that for any spacetime metric the spacetime coordinates describing where an event takes place is its spacetime position. Let's also call the derivative of spacetime position with respect to proper time the spacetime velocity, and the derivative of spacetime velocity with respect to proper time the spacetime acceleration. We can also refer to the spacetime rest mass multiplied by the spacetime acceleration the spacetime force.

In a spacetime, in which the spacetime interval between any two events is $\Delta s^2 = \Delta x_1^2 + \Delta x_2^2 + \Delta x_3^2 + \ldots + \Delta x_n^2$ for any two events there is a reference frame, for which both events happen simultaneously, as well as other reference frames for which they are not simultaneous. Also a single world line can be treated as multiple world lines that have a certain average spacetime velocity, but for which the spacetime velocity of each individual component world line can be anything. This means that for World Line A at point A, for every point on World Line B there is some component world line for World Line A, for which the point now. This implies that if World Line A, and World Line B are both world lines for electric charges, then the entire electrically charges World Line for B must be taken into account in order to find the electric spacetime force acting on World Line A at a particular point. This also implies that the electric spacetime force from an infinite line in n dimensions must have the same function as the electric spacetime force from a point in n-1 dimensions.

In order to approximate the force from an infinite line, I approximated the force from a very long line using the a Riemann Sum. I approximated the integral $\int_{-500}^{500} \left(\frac{y}{\sqrt{x^2+y^2}} \frac{\cos cos \sqrt{x^2+y^2} + \sqrt{x^2+y^2} \sin sin \sqrt{x^2+y^2}}{\sqrt{x^2+y^2}^3} \right) dy$ using a rectangle width of 0.01, and every whole number multiple of 0.01 between 0.01 and 29.99. Below is a chart from the calculation, in which I divided the integral of the supposed force from points and the integral of points divided by points the force from a line



This part shows that neither one is a whole number multiple of the other, which is what called into question whether naively changing $-\frac{\cos\cos(br) + br\sin\sin(br)}{r^2}$ to $-\frac{\cos\cos(br) + br\sin\sin(br)}{r^3}$ would work for a spacetime, in which the spacetime interval between two events would be $\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 + \Delta w^2 + \Delta u^2$, as if it did then I would expect the y value in the chart to be a constant or for any deviations from a constant to be small enough to explain by measurement errors. I found that one issue I had was that I knew how to use numerical integration to check whether a definite integral of a certain vector function would tend towards another function, but didn't know how to find what vector functions integral would tend towards a target function.

I also read that if photons have mass then the equation for the force between two electric charges would be $F_e = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2} e^{-br}(1 + br)$. This was also more complex than what I would have expected for the equation for the force between two electric charges given massive photons as what I would have expected based on photons having a half-life would have been an exponential decay function multiplied by an inverse square function. I wondered what the force law for the force between two electric charges might be in the case of massive photons in higher dimensions.

I posted a question about what the force law would be for the electric force between two electric charges in the case of massive photons in higher dimensions. The unexpected answer I got was that the equation for the electric potential energy between two electric charges, in natural units, would be $U_e = \frac{Q_1 Q_2 (m_\gamma r)^{\frac{d}{2}-1} K_{\frac{d}{2}-1}(m_\gamma r)}{(2\pi)^{\frac{d}{2}} r^{d-2}}$ with d being the number of dimensions, m_γ being the photon mass, r being the distance between the two electric charges, Q_1 and Q_2 being the two electric charges and $K_n(x)$ being the modified Bessel Function of the Second Kind. Because force is the derivative of potential energy with respect to distance, this would mean that taking the derivative of the last equation would give the force equation. Seeing as the above equation was for the case, in which the spacetime interval between two events is $\Delta s^2 = \Delta x_1^{\ 2} + \Delta x_2^{\ 2} + \Delta x_3^{\ 2} + \ldots - \Delta x_n^{\ 2}$ I conjectured that for the case, in which the spacetime interval between two events is $\Delta s^2 = \Delta x_1^{\ 2} + \Delta x_2^{\ 2} + \Delta x_3^{\ 2} + \ldots + \Delta x_n^{\ 2}$ the equation for the potential energy between two electric charges might be

$U_e = \frac{Q_1 Q_2 (m_\gamma r)^{\frac{d}{2}-1} Y_{\frac{d}{2}-1}(m_\gamma r)}{(2\pi)^{\frac{d}{2}} r^{d-2}}$. Also $\frac{\partial K_n(x)}{\partial x} = -\frac{1}{2}\left(K_{n-1}(x) + K_{n+1}(x)\right)$ and

$\frac{\partial Y_n(x)}{\partial x} = \frac{1}{2}\left(Y_{n-1}(x) - Y_{n+1}(x)\right)$.

$$\frac{(r)^{\frac{4}{2}-1} Y_{\frac{4}{2}-1}(r)}{(2\pi)^{\frac{4}{2}} r^{4-2}} = \frac{(r)^{2-1} Y_{2-1}(r)}{(2\pi)^2 r^2} = \frac{(r)^1 Y_1(r)}{(2\pi)^2 r^2} = \frac{Y_1(r)}{(2\pi)^2 r}.$$

$\frac{\partial\left(\frac{Y_1(r)}{r}\right)}{\partial r} = \frac{\left(\frac{\partial Y_1(r)}{\partial r}\right) r - Y_1(r)\left(\frac{\partial r}{\partial r}\right)}{r^2} = \frac{\frac{1}{2}\left(Y_0(r) - Y_2(r)\right) r - Y_1(r)}{r^2}$ and putting this into Wolfram Alpha I find

that it can be expressed as $-\frac{Y_2(r)}{r}$. $\frac{(r)^{\frac{3}{2}-1} Y_{\frac{3}{2}-1}(r)}{(2\pi)^{\frac{3}{2}} r^{3-2}} = \frac{(r)^{\frac{1}{2}} Y_{\frac{1}{2}}(r)}{(2\pi)^{\frac{3}{2}} r} = \frac{\left(Y_{\frac{1}{2}}(r)\right)}{\sqrt{2\pi}^3 \sqrt{r}}$ and

$Y_{\frac{1}{2}}(x) = -\frac{(\mathrm{coscos}\, x)\sqrt{2}}{\sqrt{\pi x}}$ so $\frac{(r)^{\frac{3}{2}-1} Y_{\frac{3}{2}-1}(r)}{(2\pi)^{\frac{3}{2}} r^{3-2}} = -\frac{(\mathrm{coscos}\,(r))}{2\pi^2 r}$ and the derivative of $-\frac{\mathrm{coscos}\, r}{2\pi^2 r}$ is

proportional to $\frac{r\mathrm{sinsin}\, r + \mathrm{coscos}\, r}{r^2}$. Also $\frac{(r)^{\frac{5}{2}-1} Y_{\frac{5}{2}-1}(r)}{r^{5-2}} = -\frac{\sqrt{2}(r\mathrm{sinsin}\, r + \mathrm{coscos}\, r)}{r^3\sqrt{\pi}}$ with its derivative

being $\frac{\sqrt{2}\left(3r - (r^2-3)\mathrm{coscos}\, r\right)}{\sqrt{\pi} r^4}$. I also found that $\frac{r^{\frac{1}{2}-1} Y_{\frac{1}{2}-1}(r)}{r^{\frac{1}{2}-1}} \propto \sin \sin r$ and $\frac{r^{\frac{1}{2}-1} K_{\frac{1}{2}-1}(r)}{r^{\frac{1}{2}-1}} \propto e^{-r}$.

$\frac{(r)^{\frac{3}{2}-1} Y_{\frac{3}{2}-1}(r)}{(2\pi)^{\frac{3}{2}} r^{3-2}} \propto \frac{e^{-r}}{r} \propto \frac{e^{-r(r+1)}}{r}$ with a derivative proportional to $\frac{e^{-r}(r+1)}{r^2}$. $\frac{(r)^{\frac{2}{2}-1} Y_{\frac{2}{2}-1}(r)}{(2\pi)^{\frac{2}{2}} r^{2-2}} \propto Y_0(r)$

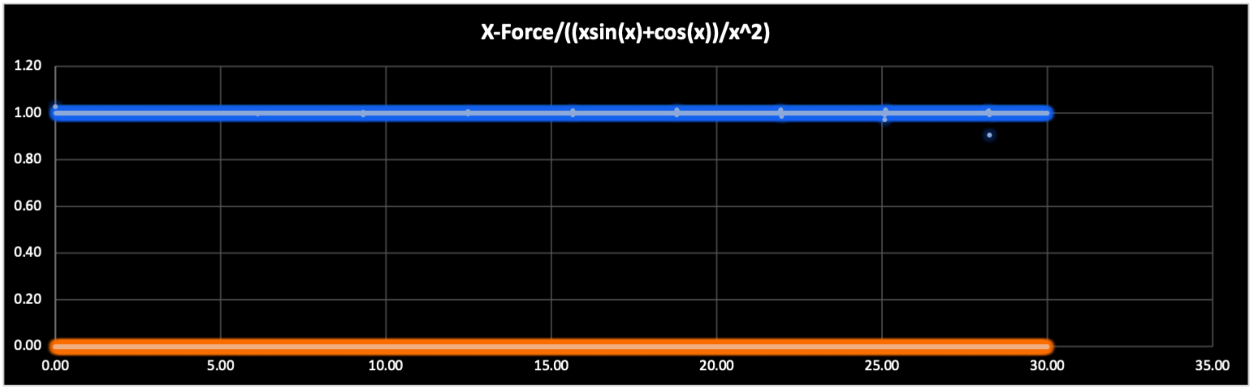
with a derivative of $\frac{1}{2}\left(Y_{-1} - Y_1\right)$ and $\frac{(r)^{\frac{2}{2}-1} K_{\frac{2}{2}-1}(r)}{(2\pi)^{\frac{2}{2}} r^{2-2}} \propto K_0(r)$ with a derivative of

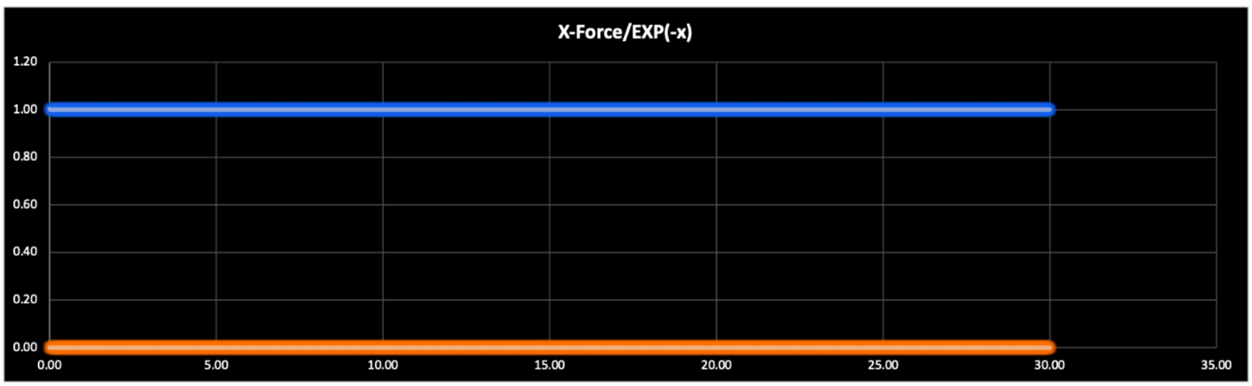
$-\frac{1}{2}\left(K_{-1} + K_1\right)$.

I did numerical integration using the same methods I mentioned for the previous chart to see if assuming a force proportional to $\frac{Y_2(r)}{r}$ for a point charge in 4 dimensions might produce a force proportional to $\frac{r\mathrm{sinsin}\, r + \mathrm{coscos}\, r}{r^2}$ for a line in 4 dimensions, and whether assuming a force proportional to $\frac{1}{2}\left(K_{-1} + K_{1}\right)$ from a point charge in 2 dimensions might

produce a force proportional to e^{-r} for a line charge in 2 dimensions.

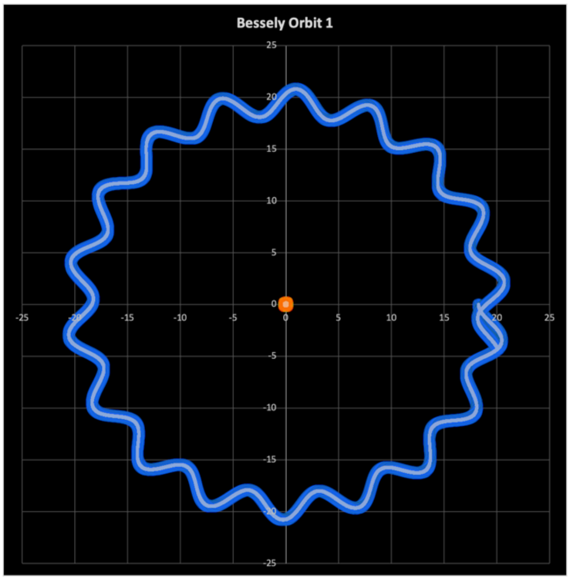
Below are the charts showing the results from this integration

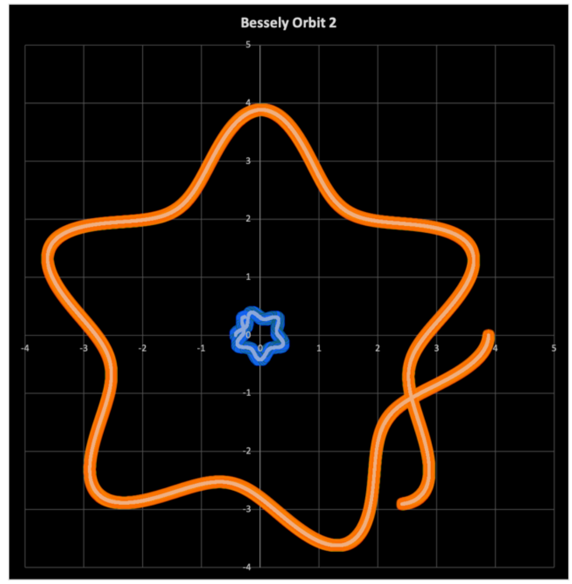


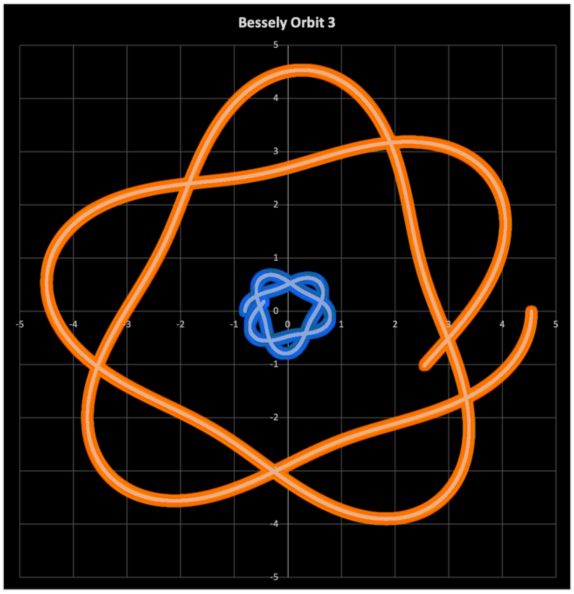


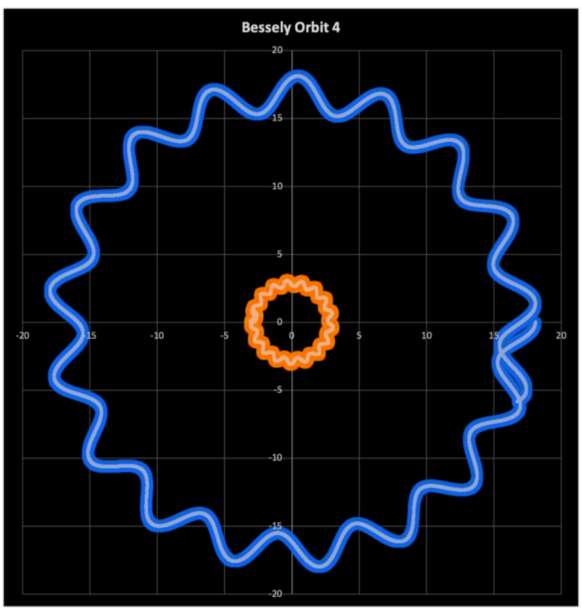
In both cases the ratio between the force found from numerical integration for a line in d dimensions is and the assumed force for a point in d-1 dimensions is nearly constant.

I ran some two body simulations using the force law $F = \frac{W_1 W_2\left(\frac{1}{2}\left(Y_0(r) - Y_2(r)\right)r - Y_1(r)\right)}{r^2}$ with W_1 and W_2 being the world charges. Below are some charts from simulations I ran using random initial conditions


Bessely Orbit 1


Bessely Orbit 2





I also was curious about how to simulate the wavefunction of a hydrogen atom using the Schrodinger equation. I saw that the time independent Schrodinger Equation is $E\Psi = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi$ with Ψ being the wavefunction, E being the Energy Level, V being the potential operator, ∇^2 being the Laplacian, m being the mass, and $\hbar$ being the reduced plank constant equal to $\frac{h}{2\pi}$. In the case of the hydrogen atom $V \propto \frac{1}{r}$. I found out that in three dimensions $\nabla^2\Psi = \frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2}$ and in one dimension$\nabla^2\Psi = \frac{\partial^2\Psi}{\partial x^2}$ and generalizing to d dimensions $\nabla^2\Psi = \frac{\partial^2\Psi}{\partial x_1^2} + \frac{\partial^2\Psi}{\partial x_2^2} + \ldots + \frac{\partial^2\Psi}{\partial x_d^2}$.

The simplest type of bound state wavefunction to simulate is one that is spherically symmetric. Along the x_1 axis r=|x_1| so part of the Laplacian in terms of r is $\frac{\partial^2}{\partial r^2}$. $\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \ldots + \frac{\partial^2}{\partial x_d^2}$ depends only on $\frac{\partial}{\partial x_1}$ and the number of dimensions, so I can derive the rest of the Laplacian, in terms, of r by setting $\frac{\partial^2\Psi}{\partial x_1^2} = 0$ and finding $\frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} + \ldots + \frac{\partial^2}{\partial x_d^2}$. When $\frac{\partial^2\Psi}{\partial x_1^2} = 0$ $\Psi = \frac{\partial\Psi}{\partial x_1}\sqrt{x_1^2 + x_2^2 + \ldots + x_d^2} + b$ so

$$\frac{\partial\Psi}{\partial x_2} = \frac{\partial\Psi}{\partial x_1}\frac{\partial\sqrt{x_1^2+x_2^2+\ldots+x_d^2}}{\partial x_2} = \frac{\partial\Psi}{\partial x_1}\frac{\partial\left(x_1^2+x_2^2+\ldots+x_d^2\right)^{\frac{1}{2}}}{\partial x_2} = \frac{\partial\Psi}{\partial x_1}x_2\left(x_1^2 + x_2^2 + \ldots + x_d^2\right)^{-\frac{1}{2}}$$

meaning

$$\frac{\partial^2\Psi}{\partial x_2^2} = \frac{\partial\Psi}{\partial x_1}\frac{\partial x_2\left(x_1^2+x_2^2+\ldots+x_d^2\right)^{-\frac{1}{2}}}{\partial x_2} = \frac{\partial\Psi}{\partial x_1}\left(\left(x_1^2 + x_2^2 + \ldots + x_d^2\right)^{-\frac{1}{2}} - x_2^2\left(x_1^2 + x_2^2 + \ldots + x_d^2\right)^{-\frac{3}{2}}\right)$$

which along the x_1 axis reduces to $\frac{1}{|x_1|}$. This means that $\nabla^2\Psi = \frac{\partial^2\Psi}{\partial r^2} + \frac{(d-1)}{r}\frac{\partial\Psi}{\partial r}$ in d dimensions, in the case of hyperspherical symmetry.